Model Answer (AV-8207)

MA/MSc. Ist Sem. Examination 2015-16

Mathematics

Discrete Mathematical Structure-I

1 (i). All the declarative sentences to which it is possible to assign one and only one of the two possible truth values are called statements.

(ii).
$$\sim(P \vee Q) \equiv \sim P \wedge \sim Q$$
$$\sim(P \wedge Q) \equiv \sim P \vee \sim Q$$

(iii). A string of formulas is defined as

(a) Any formula is a string of formulas.

(b) If $\alpha$ and $\beta$ are string of formulas, then $\alpha, \beta$ and $\beta, \alpha$ are strings of formulas.

(c) Only those strings which are obtained by steps (a) and (b) are strings of formulas, with the exception of the empty string which is also a string of formulas.

(iv). Let $(M, *, e_m)$ and $(T, \Delta, e_T)$ be any two monoids. A mapping $g: M \to T$ such that for any two elements $a, b \in M$
$$g(a*b) = g(a) \,\Delta\, g(b)$$
and
$$g(e_m) = e_T$$
is called a monoid homomorphism.

(v). Let $V$ denote a nonempty set of symbols. The set of strings over $V$ is denoted by $V^*$ and the set of nonempty strings by $V^+ = V^* - \{\Lambda\}$. Then $V^+$ is a semigroup with respect to concatenation operation. This semigroup $(V^+, \circ)$ is called a free semigroup.

(vi). Let $(L, \leq)$ be a lattice. For any $a, b, c \in L$

$$a \leq c \quad \text{iff} \quad a \oplus (b * c) \leq (a \oplus b) * c$$

(vii). A lattice $(L, *, \oplus, 0, 1)$ is said to be a complemented lattice if every element of $L$ has at least one complement.

(viii). Let $(B, *, \oplus, ', 0, 1)$ and $(P, \cap, \cup, \bar{\ }, \alpha, \beta)$ be two Boolean algebras. A mapping $f: B \to P$ is called a Boolean homomorphism if all the operations of the Boolean algebra are preserved, ie. for any $a, b \in B$

$$f(a * b) = f(a) \cap f(b), \quad f(a \oplus b) = f(a) \cup f(b)$$

$$f(a') = \overline{f(a)}, \quad f(0) = \alpha, \quad f(1) = \beta.$$

(ix). Two Boolean forms $\alpha(x_1, x_2, \cdots, x_n)$ and $\beta(x_1, x_2, \cdots x_n)$ are called equivalent if one can be obtained from the other by a finite number of applications of the identities of a Boolean algebra.

(x). A context-sensitive grammar contains only productions of the form $\alpha \to \beta$ where $|\alpha| \leq |\beta|$.

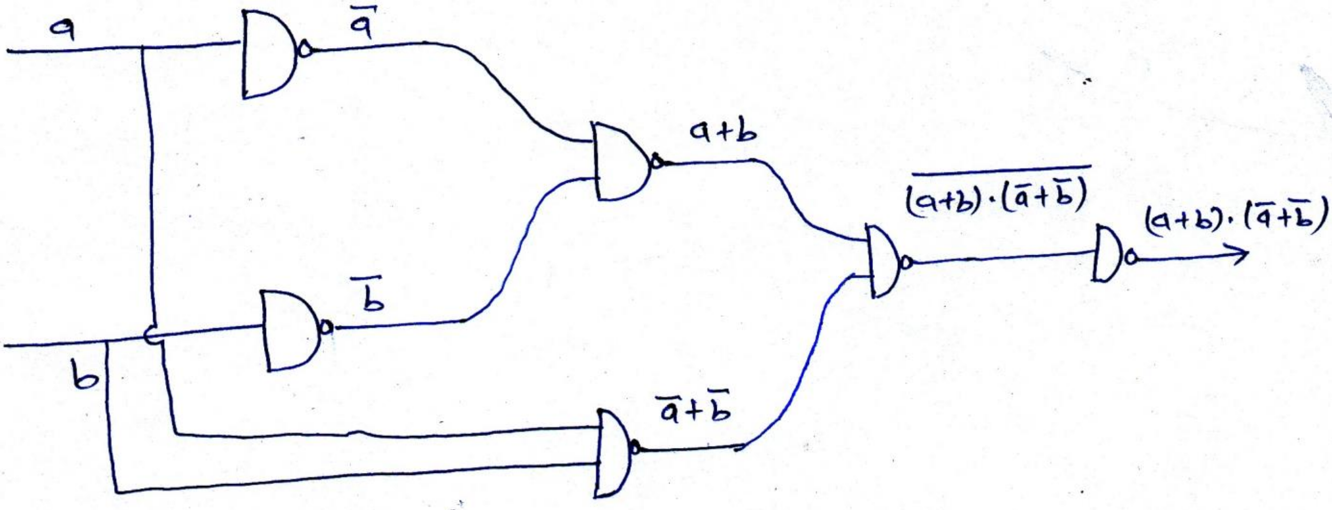
2 (a). By using the equivalence of formulas, the students have to show that

$$(P \to (Q \to R)) \to ((P \to Q) \to (P \to R)) \text{ is equivalent to } T.$$

(b). By using the equivalence of formulas, the principal disjunctive normal form is obtained as

$$(\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q).$$

3(a).




(b). The students have to show that the premises

$P \to Q,\ P \to R,\ Q \to \neg R,\ P$

will imply F (contradiction).

4(a). Let $g$ be a semigroup homomorphism from $(S, *)$ to $(T, \Delta)$. Let $a \in S$ is an idempotent element, then

$$a * a = a$$
$$g(a*a) = g(a)$$
$$g(a) \,\Delta\, g(a) = g(a)$$

Hence $g(a)$ is idempotent element.

Also let $a, b \in S$ commute with each other

i.e. $a * b = b * a$

then $g(a*b) = g(b*a)$

$$g(a) \,\Delta\, g(b) = g(b) \,\Delta\, g(a).$$

then $g(a)$ and $g(b)$ commute with each other.

(b). Let $(S, *)$ be a finite semigroup and $a \in S$.

then $a^n \in S \quad \forall n \in \mathbb{N}$

but $S$ is finite, so $\exists$ $r$ & $s \in \mathbb{N}$ s.t. $s > r$ (ie $s = r+k$)

and $a^r = a^s$

$a^r = a^{r+k}$

then $a^{2r+k} = a^r . a^{r+k} = a^r . a^r = a^{2r}$

Similarly $a^{mr+k} = a^{mr} \quad \forall m \in \mathbb{N}$.

Again $a^{mr+2k} = a^{mr+k} . a^k = a^{mr} . a^k = a^{mr+k} = a^{mr}$

Similarly $a^{mr+nk} = a^{mr} \quad \forall n \in \mathbb{N}$.

Put $m = k$ and $n = r$, we get

$a^{2kr} = a^{kr}$

ie $(a^{kr})^2 = a^{kr}$

Put $y = a^{kr}$ then

$y^2 = y$

ie $y = a^{kr} \in S$ is an idempotent element.

5(a). Let $(L, \leq)$ be a lattice and $a, b, c \in L$.

The students have to prove

$$b \leq c \implies \begin{cases} a * b \leq a * c \\ a \oplus b \leq a \oplus c \end{cases}$$

(b).

Since $a * b \leq a \leq a \oplus c$

and $a * b \leq b \leq b \oplus d$

then $a * b \leq (a \oplus c) * (b \oplus d) \quad — (1)$

Also $c * d \leq c \leq a \oplus c$

and $c * d \leq d \leq b \oplus d$

then $c * d \leq (a \oplus c) * (b \oplus d)$ —— (2)

From (1) & (2)

$$(a * b) \oplus (c * d) \leq (a \oplus c) * (b \oplus d).$$

6. Lattice as a poset.

A lattice is a partially ordered set $(L, \leq)$ in which every pair of elements $a, b \in L$ has greatest lower bound and a least upper bound.

Lattice as algebraic structure

A lattice is an algebraic system $(L, *, \oplus)$ with two binary operations $*$ and $\oplus$ on $L$ which are both, commutative, associative and satisfy the absorption laws.

The students have to obtain the properties of definiton 2 from def. 1 and also from def. 1 to def. 2.

7.(a). The sum of product canonical form is obtained as

$$x_1' x_2' x_3 x_4 + x_1' x_2' x_3 x_4' + x_1' x_2' x_3' x_4 + x_1' x_2' x_3' x_4'$$
$$+ x_1 x_2 x_3 x_4 + x_1' x_2 x_3 x_4 + x_1 x_2' x_3 x_4$$

(b). Simplified expression is as

$$(a * b') \oplus (b' * c)$$

8. Students have to discuss Context-sensitive grammar, Context-free grammar, regular grammar and free grammar.